EXERCICES SUR LES NOMBRES COMPLEXES

Exercice 4

$z_1 = 5\sqrt{2}(1+i)$ *et* $z_2 = -5(1+i\sqrt{3})$. Posons $\arg z_1 = \theta_1(2\pi)$ et $\arg z_2 = \theta_2(2\pi)$

1) Module et arguments des nombres complexes: $z_1, z_2, \overline{z}_1, \dfrac{1}{z_1}$

$|z_1| = |5\sqrt{2}(1+i)| = 5\sqrt{2}|1+i| = 5\sqrt{2}\times\sqrt{2} = 10$

On a $\cos\theta_1 = \sin\theta_1 = \dfrac{5\sqrt{2}}{10} = \dfrac{\sqrt{2}}{2}$ *donc* $\theta_1 = \dfrac{\pi}{4}(2\pi)$.

$|z_2| = |-5(1+i\sqrt{3})| = 5|1+i\sqrt{3}| = 10$.

$\cos\theta_1 = \dfrac{-5}{10} = \dfrac{-1}{2}$ *et* $\sin\theta_2 = \dfrac{-5\sqrt{3}}{10} = \dfrac{-\sqrt{3}}{2}$ *donc* $\theta_2 = -\dfrac{2\pi}{3}(2\pi)$.

$|\overline{z}_1| = |z_1| = 10$; $\arg\overline{z}_1 = -\arg z_1 = -\dfrac{\pi}{4}(2\pi)$. $\left|\dfrac{1}{z_1}\right| = \dfrac{1}{|z_1|} = \dfrac{1}{10}$; $\arg\dfrac{1}{z_1} = -\arg z_1 = -\dfrac{\pi}{4}(2\pi)$

2) Soit Z le nombre complexe tel que : $z_1 Z = z_2$

$Z = \dfrac{z_2}{z_1}$ *d'où* $|Z| = \left|\dfrac{z_2}{z_1}\right| = 1$. $\arg Z = \arg\dfrac{z_2}{z_1} = \arg z_2 - \arg z_1(2\pi)$

$\arg Z = -\dfrac{2\pi}{3} - \dfrac{\pi}{4}(2\pi)$ $\arg Z = \dfrac{-11\pi}{12}(2\pi) = \dfrac{13\pi}{12}(2\pi)$.

On en déduit la forme trigonométrique de Z : $Z = \cos\left(\dfrac{13\pi}{12}\right) + i\sin\left(\dfrac{13\pi}{12}\right)$

Forme algébrique de Z:

$Z = \dfrac{z_2}{z_1} = \dfrac{-5(1+i\sqrt{3})}{5\sqrt{2}(1+i)} = \dfrac{-1}{\sqrt{2}}\times\dfrac{1+i\sqrt{3}}{1+i} = \dfrac{-1}{\sqrt{2}}\dfrac{(1+i\sqrt{3})(1-i)}{(1+i)(1-i)} = \dfrac{(-1-\sqrt{3})+i(1-\sqrt{3})}{2\sqrt{2}}$

$Z = \dfrac{-1-\sqrt{3}}{2\sqrt{2}} + i\dfrac{1-\sqrt{3}}{2\sqrt{2}}$ $Z = \dfrac{-\sqrt{2}-\sqrt{6}}{4} + i\dfrac{\sqrt{2}-\sqrt{6}}{4}$

3) En égalisant les formes algébrique et trigonométrique de Z on a

$Z = \dfrac{-\sqrt{2}-\sqrt{6}}{4} + i\dfrac{\sqrt{2}-\sqrt{6}}{4} = \cos\left(\dfrac{13\pi}{12}\right) + i\sin\left(\dfrac{13\pi}{12}\right)$

on en déduit que : $\cos\left(\dfrac{13\pi}{12}\right) = \dfrac{-\sqrt{2}-\sqrt{6}}{4}$ et $\sin\left(\dfrac{13\pi}{12}\right) = \dfrac{\sqrt{2}-\sqrt{6}}{4}$